

Variable Force

Recall that:

Fact —

$$\text{Force} = \frac{d}{dt}(\text{momentum}) \quad (\text{N2})$$

$$\text{Impulse} = \Delta \text{momentum}$$

$$\text{Impulse} = \text{Force} \times \text{time}$$

Integrating both sides, we can obtain:

Fact (Impulse of a variable force) —

$$\text{Impulse} = \int \text{Force } dt = \Delta \text{momentum}$$

Example

A pebble of mass 0.2 kg is moving on a smooth horizontal sheet of ice. At time t seconds (where $t \geq 0$) a horizontal force of magnitude $2t^2$ N and constant direction acts on the pebble. When $t = 0$ the pebble is moving in the same direction as the force and has speed 6 ms^{-1} . When $t = T$ the pebble has speed 36 ms^{-1} .

Calculate the value of T .

Example

A particle, A , of mass 500 g, is acted on by a variable force F N, which is defined as:

$$F = \begin{cases} 0.3t^2 + 0.5t & \text{for } 0 \leq t \leq 3 \text{ s} \\ t + 1.2 & \text{for } 3 < t \leq 5 \text{ s} \end{cases}$$

Find the speed of A after 4 seconds if the speed is 2 ms^{-1} when $t = 0$.

Vector Notation

Example

A toy sailing boat of mass 800 g is blown along by a constant wind, which produces a force of $(\mathbf{i} + 2\mathbf{j})\text{N}$ on the boat. If the boat is initially at rest, find its velocity after 10 s.

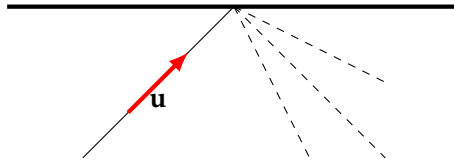
Example

A particle A of mass 2 kg is moving with velocity $(2\mathbf{i} + 3\mathbf{j}) \text{ms}^{-1}$ when it collides with particle B , of mass 1.5 kg, which is moving with velocity $(\mathbf{i} + 2\mathbf{j}) \text{ms}^{-1}$. If the particles coalesce during the collision:

- (a) find the velocity of the combined particle after the collision
- (b) find the total loss of energy as a result of the collision.

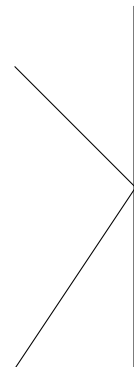
Oblique Impacts

Consider an object colliding with a wall - where does it go?



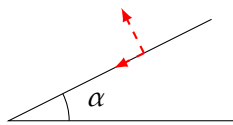
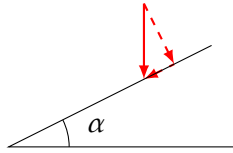
Example

A smooth sphere S is moving on a smooth horizontal plane with speed u when it collides with a smooth fixed vertical wall. At the instant of the collision the direction of motion of S makes an angle of 60° with the wall. The coefficient of restitution between S and the wall is $\frac{1}{4}$. Find the speed of S immediately after the collision

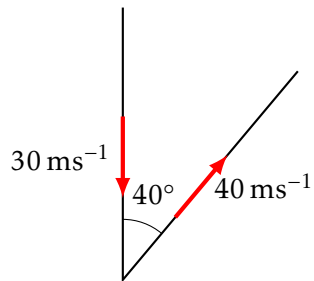


Example

A smooth ball is falling vertically. The ball strikes a smooth plane which is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{1}{2}$. Immediately before striking the plane the ball has speed 5 ms^{-1} . The coefficient of restitution between the ball and the plane is $\frac{1}{2}$. Find the speed of the ball immediately after the impact.

**Example**

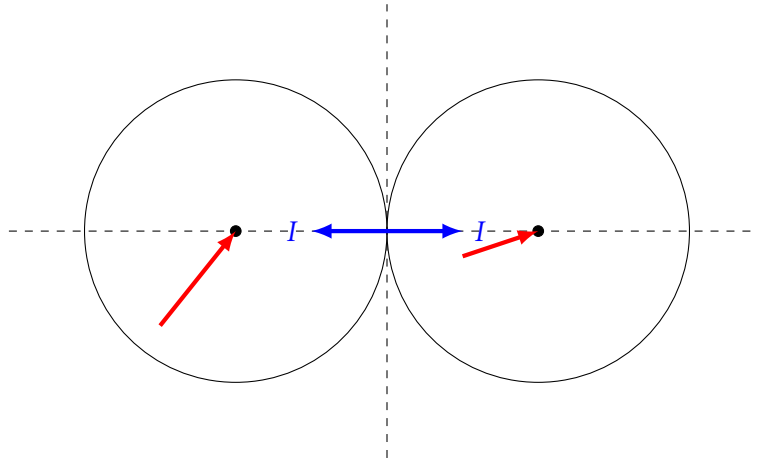
A cricket ball of mass 160 g is travelling horizontally at a speed of 30 ms^{-1} when it is hit by a cricket bat. It leaves the bat, horizontally, at a speed of 40 ms^{-1} travelling at 40° to the line of its original path, as shown in the diagram. What is the magnitude of the impulse of the bat on the ball?



Oblique Collision of Two Spheres

Example

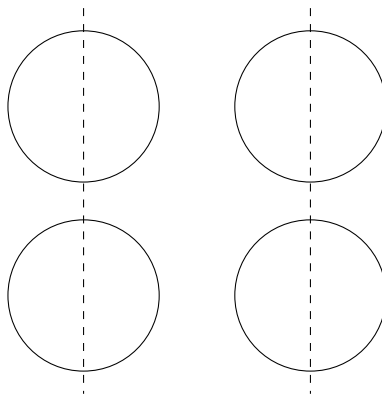
A sphere, A of mass 2 kg , is moving on a smooth horizontal surface with velocity $(4\mathbf{i} + 5\mathbf{j})\text{ms}^{-1}$ when it collides with a second sphere, B , of equal size and mass 3 kg , moving on the same surface with velocity $(3\mathbf{i} + \mathbf{j})\text{ms}^{-1}$. The line of the centres of the two spheres at the moment of collision is parallel to the \mathbf{i} -direction and the coefficient of restitution between the two spheres is 0.8 . Find the velocities of A and B immediately after the collision.



Example

A smooth sphere A of mass 5 kg is moving on a smooth horizontal surface with velocity $(2\mathbf{i} + 3\mathbf{j})\text{ m s}^{-1}$. Another smooth sphere B of mass 3 kg and the same radius as A is moving on the same surface with velocity $(4\mathbf{i} - 2\mathbf{j})\text{ m s}^{-1}$. The spheres collide when their line of centres is parallel to \mathbf{j} . The coefficient of restitution between the spheres is $\frac{3}{5}$.

Find the velocities of both spheres after the impact.



Example

Two small smooth spheres A and B have equal radii. The mass of A is $2m$ kg and the mass of B is $3m$ kg. The spheres are moving on a smooth horizontal plane and they collide. Immediately before the collision the velocity of A is $(5\mathbf{j})\text{ms}^{-1}$ and the velocity of B is $(3\mathbf{i} - \mathbf{j})\text{ms}^{-1}$. Immediately after the collision the velocity of A is $(3\mathbf{i} + 2\mathbf{j})\text{ms}^{-1}$. Find

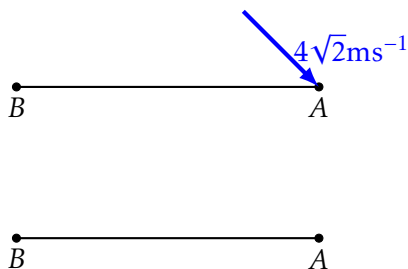
- (a) the speed of B immediately after the collision
- (b) a unit vector parallel to the line of centres of the spheres at the instance of the collision

Impulsive Tension in Strings

Example

In the diagram, the particles A and B , of masses 2.5 kg and 1.5 kg respectively, are joined by a light, inextensible string that is straight but not under tension. Particle B is initially at rest and particle A is projected at an angle of 45° to the string with speed $4\sqrt{2}\text{ ms}^{-1}$.

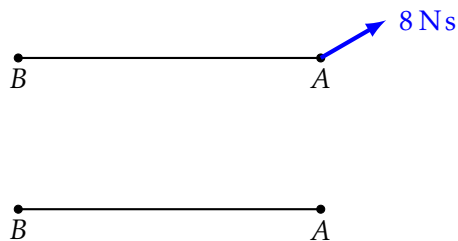
Find the speed of B immediately after the string goes taut. Find the impulse in the string.



Example

A light, inextensible string has particles A and B , each of mass 400 g , attached to either end. The string and particles rest on a smooth table. The particle at A is hit with an impulse of 8 N s at an angle of 30° to the string, which is straight but not taut and at rest.

What is the velocity of B after the impulse at the instant the string becomes taut?



Example (OCR June 2019 - Q6)

Two particles A and B , of masses m kg and 1 kg respectively, are connected by a light inextensible string of length d m and placed at rest on a smooth horizontal plane a distance of $\frac{1}{2}d$ m apart. B is then projected horizontally with speed v m s⁻¹ in a direction perpendicular to AB .

- (a) Show that, at the instant that the string becomes taut, the magnitude of the instantaneous impulse in the string, I N s, is given by

$$I = \frac{\sqrt{3}mv}{2(1+m)}.$$

[5]

- (b) Find, in terms of m and v , the kinetic energy of B at the instant after the string becomes taut. Give your answer as a single algebraic fraction. [3]

- (c) In the case where m is very large, describe, with justification, the approximate motion of B after the string becomes taut. [2]